Swami Ramanand Teerth Marathwada University Nanded, Vishnupuri Nanded (M.S.), India.

"Mathematical Models for Various Diseases Through Fractional Differential Equations"

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Submitted by

Mr. Shejul Sachin Vishnurao

N.E.S. Science College, Nanded

Email: shejulsachin222@gmail.com

Under The Supervision of

Dr. V.C. Borkar

Associate Professor & Head,

Department of Mathematics & Statistics,

Yeshwant Mahavidyalaya, Nanded

(M.S.) India

Email: borkarvc@gmail.com

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1 Abstract:

In this work, we apply the fractional differential transform approach to approximate a fractional-order mathematical model of numerous illnesses and their consequences. This research study is being carried out with the goal of obtaining an analysis based on several types of commonly used ordinary differential equations models connected to illnesses. The problem is mathematically modelled as a system of nonlinear fractional differential equations. To demonstrate the answer, graphical data are provided and quantitatively examined. In the section on viscoelasticity, we discuss several common models that generalise conventional models by substituting fractional terms for classical components. We conclude that the suggested system might be used as a kernel for comprehending, analysing, and computing solutions for a variety of problems. In this work, we will try to formulate and analyze a new mathematical model for various diseases with isolated class in fractional order.

2 Introduction to proposed research topic:

Mathematical modelling of the transmission of infectious illnesses has piqued the interest of academics from a variety of fields. The modelling is essential because it has become a key tool in understanding disease dynamics and in decision-making about fast intervention programmes in controlling and preventing the development of these illnesses. Until the advent of mathematical tools, real-world issues were represented using an abstract model. These mathematical models include fundamental variables and connections that correlate to observable and recorded occurrences and behaviour. Because the theory of differential equations is insufficient to describe and model many real-world and complicated situations, the vital role of fractional differential equations in modelling these challenges has emerged in recent years. [1]

One of the primary applications of mathematical modelling is the study of disease propagation. There has been a tremendous amount of effort put into studying the dynamics of infectious illness and numerous generalisations of epidemic models that are available in the literature. Recently, there has been a surge of interest in epidemic models with temporal delays and fractional-order derivatives. Real-world application difficulties need the use of fractional-order differential equations (FODE). These equations are extensions of ordinary differential equations and are said to have originated with L'Hopital's inquiry to Leibniz in a report letter near the end of the 17th century. Many renowned mathematicians contributed to the Theory of Fractional Calculus, including Liouville, Riemann, Fourier, and Abel. FODE models provide benefits over traditional ODE models.Historically, the study of a so-called Susceptible-Infected-Recovered (SIR) model goes back to the work by Kermack and McKendrick. Since then, this model has been widely studied to predict the dynamics of different infectious diseases. [2]

Some definitions about fractional derivatives are given as follows:

Riemann Liouville Fractional Derivative :

For $\alpha \in [n-1, n)$,the Riemann Liouville's α - derivative [3] of f is

$$D_a^{\alpha}\{f(t)\} = \frac{1}{\Gamma(n-\alpha)}\frac{d^n}{dt^n}\int_a^t \frac{f(x)}{(t-x)^{(\alpha-n+1)}}dx$$

Caputo's Fractional Derivative :

For $\alpha \in [n-1, n)$,the Caputo's α - derivative [3] of f is

$$D_a^{\alpha}\{f(t)\} = \frac{1}{\Gamma(n-\alpha)}\int_a^t \frac{f^{(n)}(x)}{(t-x)^{(\alpha-n+1)}}dx$$

We can introduce the notion of fractional difference in the sense of Caputo for the different notions of fractional sums

3 Purpose of carrying out proposed research work:

Mathematical models based on ordinary differential equations of integer order have been shown to be useful in understanding biological system dynamics. Most biological systems, on the other hand, have memory or aftereffects in their behaviour. The use of fractional-order differential equations to model these systems provides greater advantages than traditional integer-order mathematical modelling, which ignores such effects. As a result, fractional calculus (i.e., calculus of integrals and derivatives of arbitrary order) has grown in popularity and significance, owing to its demonstrated applications in a wide range of scientific and engineering domains.In recent study, it has been shown that fractional-order operators, in comparison to traditional ordinary operators, have a higher capacity to represent the non-local and chaotic behaviour of numerous illnesses such as measles, TB, HIV, typhoid, zika, yellow fever, and rubella.So we have to formulate new model or redevelop the existing models for such infectious diseases by changing the affecting factors to obtain the stability of different methods that we are using and we have to analyse the solutions of fractional order mathematical models using Different Methods.

4 Review of literature and devlopment in the relevant subject:

The Differential Transform Approach (DTM) [4] is a numerical method for electric circuit analysis created by Zhou in 1986. It is based on George Adomain's Adomian Decomposition Method (ADM), which he utilised from the 1970s until the 1990s. Linear and non-linear differential equations have been solved using DTM. ADM and DTM have been proven to solve a broad class of linear and non-linear, ordinary, partly deterministic, or stochastic differential equations with approximate solutions that converge swiftly to exact solutions effectively, simply, and correctly. The work's primary objective is to apply

the ADM and DTM to ordinary differential equations (ODE) and partial differential equations (PDE) (PDE). This method currently has a well-established reputation in the literature for providing the answer in a timely manner.

The Adomain Decomposition Method (ADM) [5] was proposed in 2002 to solve fractional differential equations, however several studies found calculating the Adomain polynomials to be extremely complex. Momani and Odibate introduced the Homotopi - Perturbation Approach (HPM) [6] to fractional differential equations in 2007, demonstrating that HPM is a viable alternative method for getting fractional differential equation solutions. Another effective strategy is the Variational Iteration Method (VIM) [7], which has been effectively used in a variety of scenarios.

The Authors Area I. , Batarfi H. [8] have used mathematical models which consider susceptible, exposed, infectious, and removed people.These models are based on differential equations which are numerically solved to provide the graphs included in the paper. First, they consider classical differential equations and secondly fractional differential equations which generalize the classical approach.Finally concluded that the integer order differential equation SEIR model gives better results than the fractional SEIR model.

M. Khalil, Hosham H.A. & Saad Rida [9] studied the behavior of vector-borne plant disease fractional order model. The influence of the fractional order derivative, which is considered as the index of memory on the vector.They argued that the fractional order models are more suitable than integer order ones in plant diseases models where memory effects are essential.

A mathematical model for the vector transmission and control of banana Xanthomonas wilt incorporating roguing of infected plants and replanting using healthy or disease free suckers was formulated by Tumwiine J. & Kweyunga Eliab Horub [10] . The steady states were obtained and their stability established.

Some Authors have applied the novel introduced Caputo–Fabrizio arbitrary order derivative to the diabetesmodel and sloved the fractional problem via HATM and Padé approximation which was discussed in [11].The HATM is a pioneering and strong computational scheme to solve noninteger order differential equations.The results of this study are very helpful for medical practitioners dealing with diabetes and related issues. Thus, They

have concluded that the CF derivative is useful in the description of physical, chemical, biological, medical, social, and engineering processes. The dynamics of a cancer model in fractional derivative is ana- lyzed by M. M. El-Dessoky, Muhammad Altaf Khan [12]. They presented a cancer model in integer case and then, with some set of suitable parameters scaling the model is reduced. The reduced model was considered and applied the Caputo-Fabrizio fractional operator.They found that there exists multiple equilibria for the cancer fractional model. The possible three equilibria that is feasible biologically were used to present the model stability.

M. R. Islam built [13] an ODE model of Bsal transmission dynamics & compared an ODE model of measles transmission dynamics with its corresponding FDE model And they made conclusion that empirical data are important for the development of infectious disease models.

Some Authors developed an integer order mathematical model of PLRV transmission dynamics which was discussed in [14] and then it is extended to fractional order.They have studied the qualitative behaviors of the model by obtaining the feasible region, positivity of the solution, equilibrium points and analyzed their local and global stability and basic reproduction number of the model.

Some Important Methods are as follows:

1)Laplace Transform of Riemann Liouville Fractional Derivative Method

2)Fractional Differential Transform Method

3)Fractional Power Series Method

4)Fractional Variational Iteration Method

5)Homotopy Analysis Method

6)Adomian Decomposition Method

5 Importance of the Study:

In Mathematics and Biology, fractional calculus is becoming a helpful and important instrument in the investigation of scientific problems. A computer model may be used to investigate the prevalence of a viral infectious illness that is frequent in babies and children.

Because of the significant incidence of this disease in different regions of the globe and the need of discovering and regulating the variables that influence its prevalence, researchers from numerous scientific fields have done a variety of studies.

a)International and National Status:

Many other scientists and researchers, such as Leibniz, Euler, Lagrance, Laplace, Fourier, Abel, Liouvielli, Grunwald, Riemann, Riesz, Miller, Ross, Igor Prodlubny, Richtmeyer, Morton, Smith, Anderson, Roache, and others, contributed to the development of fractional differential equations and fractional calculus. Indian's also contributed remarkable in the development of fractional calculus like R.K. Sexana, V.Srivastava, K.N. Rai, S. Das, K. Gupta, S. Saha Roy, V.B.L. Chawrasia, D. Kumar, Pradeep Kumar, D. Bahuguna etc.

b) Scope of the Study:

Fractional calculus has been used in nearly every discipline of science, including biology, over the previous decade.The risk of spread infectious diseases in the environment is always one of the main threats to the life of living organisms. This point can be assumed as a clear proof of the importance of studying such problems from various aspects such as computational mathematical models. In this contribution, we examine a mathematical model to investigate the prevalence of an infectious disease in a prey and predator system, including three subpopulations through a fractional system of nonlinear equations.

Many application of fractional calculus can be found in Biology, nonlinear biological systems and astrophysics. There are few different methods for obtaining the solution of fractional differential equation (FDE), therefore, we develop new different methods to obtain their solutions upto the length. We will study the powerful mathematical software Mathematica to simulate the solutions graphically. Further we will study fractional wave equations, fractional differential equations and fractional partial differential equations. We will develop iterative and finite difference schemes to solve them. Solutions of these equations will be simulated graphically.

6 Research Methodology:

The following steps will be including in the methodology.

1) To look up the books on applications of fractional differential equation and books related to Mathematical Models.

2) To work on hybrid methods for obtaining approximate solutions of fractional differential models on a slew of biological conditions .

3) To discuss the stability and convergence of analytical and numerical methods which are used to analyse models for various diseases.

7 Expected Results:

1) We will try to analyze the solutions obtained by analytical methods for fractional differential equations relating to a variety of children's diseases.

2) We will try to find solutions of fractional order model related to biological aspects,health related diseases by using hybrid methods.

3) We will try to find better Hybrid Methods which will give approximate soltions of mathematical Models.

4) To obtain approximate fractional differential solutions using both ancient and modern techniques, and to visualise them using tools such as Mathematica, Matlab, Python, Scilab, and Mathcad.

8 Plan of work:

First year plan:

1) To read the reference books of applications of fractional differential equation related to Various Diseases.

2) To investigate the number of reputed research papers of applications of Fractional calculas .

3) To study softwares like Python,Mathematica and Matlab.

Second year plan:

1) To study fractional calculus.

2) To study relations in between fractional derivative and applicaions related to various diseases like tuberculosis, HIV, typhoid, zika, yellow fever.

3) To develop and analyze the fractional order Mathematical Model based on Biological Aspects.

4) To visualize the solutions by graphs using Mathematica, Matlab and discuss the nature.

5) To think about iterative approaches,stability and convergence.

Third year plan:

1) To develop Hybrid methods for finding solutions.

2) To analysis the different methods for solving fractional order Models on Various Diseases.

3) To publish our work in reputed journals.

4) Submission of completed work to concerned authority.

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Mr. Sachin Vishnurao Shejul
Research Scholar

Dr. V.C. Borkar
Research Guide